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Presente.-

Estimado/s Autor/es:

En nombre del Comité Organizador del XI Congreso Internacional de Métodos Numéricos en Ingeniería y Ciencias Aplicadas, tenemos el placer de informarle/s que su trabajo identificado con el código **CIM_158** y titulado "**COMPARISON OF PARTICLE DISPERSION OBTAINED WITH SIMPLE STOCHASTIC LAGRANGIAN MODELS AND LES IN PIPE FLOWS**" ha sido arbitrado y aceptado para su presentación en el CIMENICS 2014.

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COMPARISON OF PARTICLE DISPERSION OBTAINED WITH SIMPLE STOCHASTIC MODELS AND LES IN PIPE FLOW

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Abstract. *A hybrid Eulerian-Lagrangian approach is employed to simulate heavy particle dispersion in turbulent pipe flow. The mean flow is provided by the Eulerian simulations developed by mean of JetCode, whereas the fluid fluctuations seen by particles are prescribed by a stochastic differential equation based on normalized Langevin. The statistics of particle velocity are compared to LES data which contain detailed statistics of velocity for particles with diameter equal to 20.4 µm. The model is in good agreement with the LES data for axial mean velocity whereas rms of axial and radial velocities should be adjusted.*

Keywords: Langevin equation, LES, inhomogeneous turbulence

1. INTRODUCTION

The study of confined turbulent flows laden with solid particles is complicated due to the anisotropy and inhomogeneity of flow and its complex interaction with solid particles. Many methods have been developed to reproduce the turbulence by solving the transient Navier-Stokes equations on a discretized domain including direct numerical simulation (DNS) and large eddy simulation (LES). Both methods are conceptually similar, except that the computational effort in LES reduced, requiring a grid to be only so fine as to resolve the largest eddies, whereas the smaller eddies are modeled, however DNS and LES are computationally expensive. An alternative less expensive is the use of the stochastic models based on a Langevin equation; in this investigation our goal is use statistics of fluid velocity calculated by LES and generate the turbulent fluctuations using Langevin equations. Finally, we will compare results for particles using Lagrangian model with LES results.

2. PARTICLE EQUATION OF MOTION

In the present study, particles density (ρ_p) and diameters (d_p) were set to 1000 kg.m^{-3} and 20.4 μm respectively. Particle motion is described by a set of ordinary differential equations for velocity and position at each time step. We have considered that a one-way coupling approach is sufficient to describe the particle-fluid interactions, the density of particles is much larger than the gas density therefore Basset forces and virtual mass can be neglected; Brownian diffusion can be neglected too (it becomes important for particles smaller than those considered in this paper) therefore only drag force is taking into account. With these simplifications the following Lagrangian equations for the particle velocity are obtained:

$$\frac{dx_{pi}}{dt} = v_{pi} \tag{1}$$

$$\frac{dv_{pi}}{dt} = -\frac{\rho}{\rho_p}\frac{3}{4}\frac{C_D}{d_p}\left|v_p - \bar{u}\right|\left(v_{pi} - \bar{u}_i\right) + g_i = -F_{di} + g_i \tag{2}$$

where: x_{pi} is the particle position, v_{pi} is the particle velocity, $\bar{u}$ is the filtered gas velocity at the particle position, d_p the particle diameter and F_{di} the drag force. The Stokes drag coefficient C_D valid for particles Reynolds number Re_p up to about 40 is computed as:

$$C_D = \frac{24}{Re_p}\left(1 + 0.15Re_p^{0.687}\right) \tag{3}$$

In the presence of turbulence, the integration of the particles paths in a LES framework requires the specification of fluctuating part of fluid velocity which is extracted from averaged fields by the stochastic normalized Langevin model that defines the fluctuating velocity field along the particle track in a turbulent pipe flow for which turbulence is inhomogeneous in wall normal (radial direction). Following the normalized Langevin equations in streamwise and radial directions of the boundary layer [1]:

$$d\left(\frac{u_1}{\sigma_1}\right) = -\left(\frac{u_1}{\sigma_1}\right)\frac{dt}{\tau_L} + \sqrt{\frac{2}{\tau_L}}d\xi_1 + \frac{\partial\left(\frac{\overline{u_1 u_2}}{\sigma_1}\right)}{\partial x_2}.\frac{dt}{1 + Stk} \tag{4}$$

$$d\left(\frac{u_2}{\sigma_2}\right) = -\left(\frac{u_2}{\sigma_2}\right)\frac{dt}{\tau_L} + \sqrt{\frac{2}{\tau_L}}d\xi_2 + \frac{\partial\sigma_2}{\partial x_2}.\frac{dt}{1 + Stk} \tag{5}$$

where $d\xi_i s$ are modeled as a series of uncorrelated Gaussian random numbers with zero mean and variance *dt.* The particle Stokes number is $Stk = \tau_p/\tau_L$ and τ_L is a Lagrangian time scale [2]

3. EULERIAN SIMULATION

The continuous phase is an incompressible fully developed turbulent flow of air (kinematic viscosity ν_g = 1.57 x 10^{-6} $m^2.s^{-1}$, density ρ = 1.3 $kg.m^{-3}$). Computation of the Eulerian velocity field has been made with the code developed by Pierce and Moin [3]; incompressible Navier Stokes equations are solved numerically in cylindrical coordinates using an implicit filtering. Time integration is based on the fractional step method [4] and a second-order Adams-Bashforth is used for advancement of the convective terms while implicit Crank-Nicholson method is applied for an update of the viscous term. The Poisson equation for pressure is solved using spectral techniques. The time step in wall units imposed by numerical stability requirements (Courant-Friedrich-Levy number: $CFL<1$) is $\Delta t^+ = 0.10$. For the pipe flow, the origin of the coordinate system is located at the pipe center and the coordinates x, r and θ correspond to the streamwise, radial and azimuthal directions respectively, however the graphics results are represented with respect to $y = R - r$ (y is positive from the wall pipe to the center). For pipe flow simulation the Reynolds number is 2152, which is based on estimated average velocity (u_b = 1.69 $m.s^{-1}$), and radius pipe (R = 0.02 m). We have used a domain of 1885 x 300 x 942 wall units in x, r, θ directions, discretized with 96 x 96 x 64 nodes. A hexahedral grid is used; points are equally spaced in streamwise and azimuthal directions, while a non-uniform discretization was used for the wall normal direction in order to obtain a finer grid next to the wall. The grid spacing in wall units is Δx^+=20 in the streamwise direction and the minimum grid spacing in wall normal direction are $\Delta y^+ = 0.8$

Periodic boundary conditions are imposed on the fluid velocity field in the axial and spanwise directions and no-slip boundary conditions are enforced at the walls. Pipe flow was initialized with a power law profile at the inlet plane respectively with random fluctuations throughout the entire domain. The flow was enforced by a constant wall shear stress. At the beginning of simulation, particles were distributed randomly over the computational domain and their initial velocity was set equal to that of fluid in the particle initial position. Periodic boundary conditions were imposed on particles in both streamwise and spanwise directions. The pipe walls are perfectly smooth and elastic collisions were assumed for particles impacting the wall. To calculate particle trajectories in the flow field, we have coupled a Lagrangian tracking routines [5] with LES flow solver. The routines use a tri-linear interpolation method to determinate the fluid velocity at particle position; with this velocity the equations of particle motion are advanced in time using a fourth order Runge-Kutta scheme. The time step used for particle tracking was chosen to be equal to the time step size used for the fluid, also two orders of magnitude less than relaxation time of particle.

4. RESULTS AND DISCUSSION

In each time step 10 particles are injected from the inlet pipe; for the results shown, all velocities and rms of velocities are normalized with the friction velocity. The Figure 1 shows axial mean velocity profiles for LES fluid and particles with Langevin model. We can see that they are essentially identical.

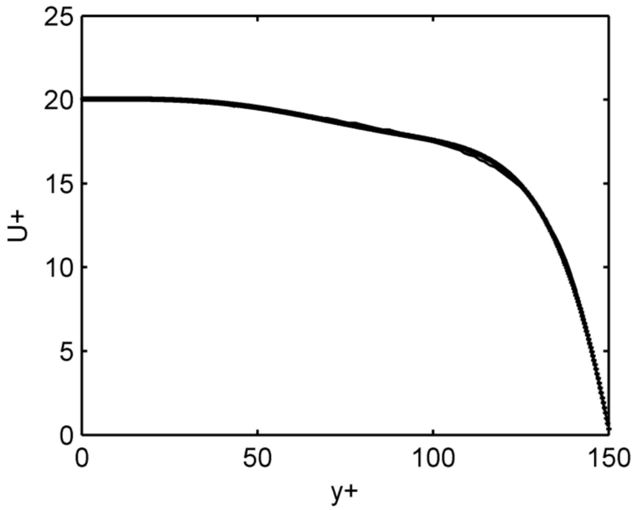

Figure 1. Mean axial velocity. (—) LES fluid; (+) Langevin model for particles

The rms values for the axial velocity for particles are shown in Figure 2.

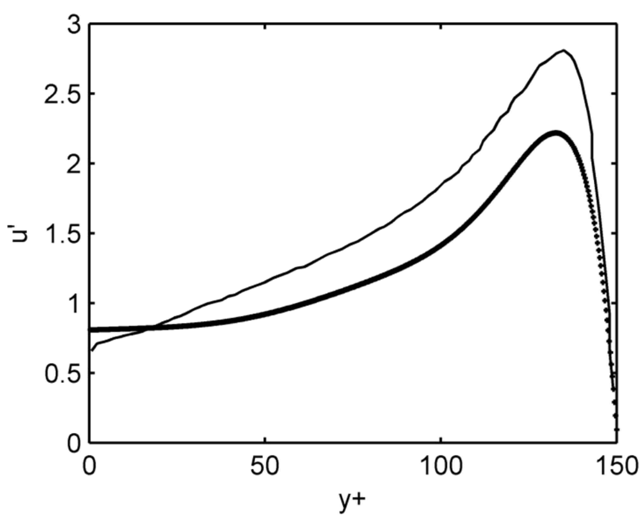

Figure 2. rms of axial velocity. (—) LES ; (+) Langevin model

The Langevin model predictions are in good agreement with LES data around pipe center. The place of maximum value is adequately predicted, however is mostly underpredicted. That behavior was also observed by Dehbi [1]. Figure 3 shows rms of radial velocity.

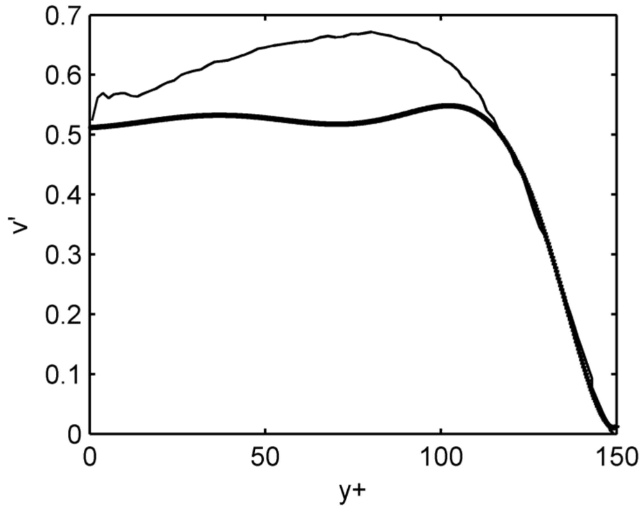

Figure 2. rms of radial velocity. (—) LES; (+) Langevin model

Both profiles match next to the center pipe whereas that up to $y^+\approx 120$ Langevin model underestimates rms radial velocities.

Differences observed in both figures as shown above can be due the assumption of Gaussian turbulent scales and Lagrangian time scales used in the normalized Langevin equation.

5. CONCLUSIONS

Normalized Langevin equations have been employed to predict particle velocity statistics in a pipe flow with anisotropic and inhomogeneous turbulence in the wall normal direction. Eulerian statistics were provided by LES performed with JetCode. The model results were compared to LES data finding good agreement in axial mean velocity profiles, the rms of velocities modeled by Langevin equations exhibit some difference with LES results so that model will be reviewed.

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